# CALCULUS III

## MATH 20550, SECTION 04

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| | | tentative: |
|---|---|---|
| Exam 1 | 20% | Sep 23 |
| Exam 2 | 20% | Oct 30 |
| Exam 3 | 20% | Nov 20 |
| Final | 30% | Dec 17 |
| Tutorials | 5% | Thursdays |
| Webassign (Homework) | 5% | |

[Make sure you go to tutorials you registered for]

← Attendence is mandatory

| | |
|---|---|
| ≥ 94 % | A |
| ≥ 84 % | A-, B+, B |
| ≥ 74 % | B-, C+, C |
| ≥ 60 % | C-, D |

Read Syllabus on Canvas

**Lectures:** MWF 11:30-12:20

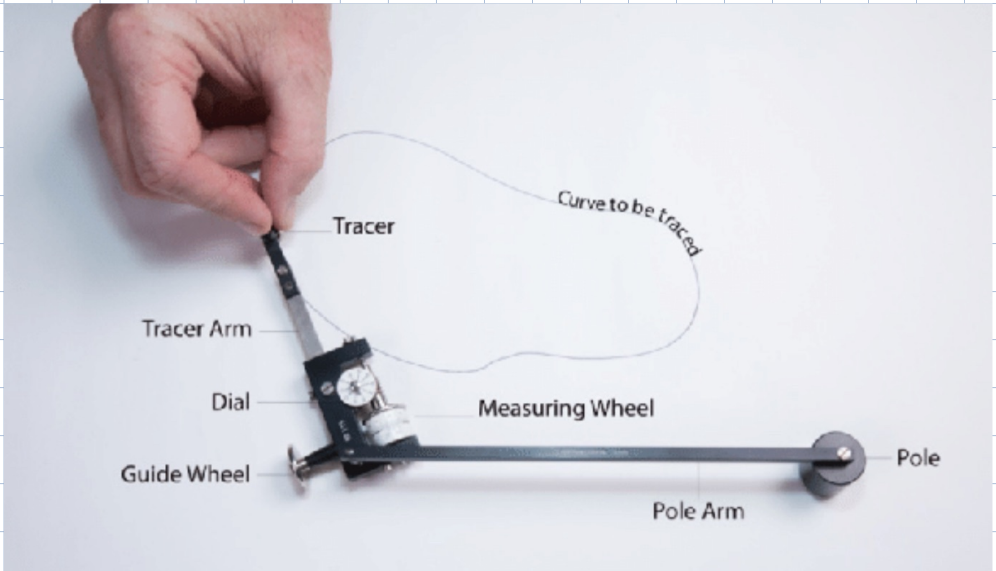
- You are expected to attend all lectures

Office Hours: Tue 2-3
126 HH Wed 2-3

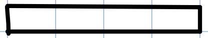
# Motivation:

## Polar Planimeter

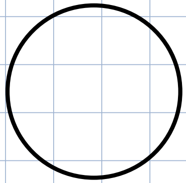
- measures the area by just going around the perimeter



Q:

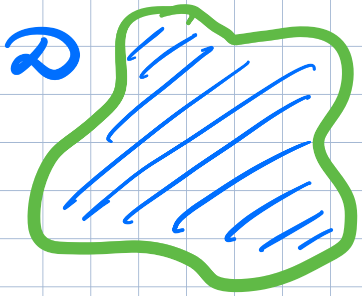


vs



A: It takes in account how the angle changes when go around.

## Greens Thm:



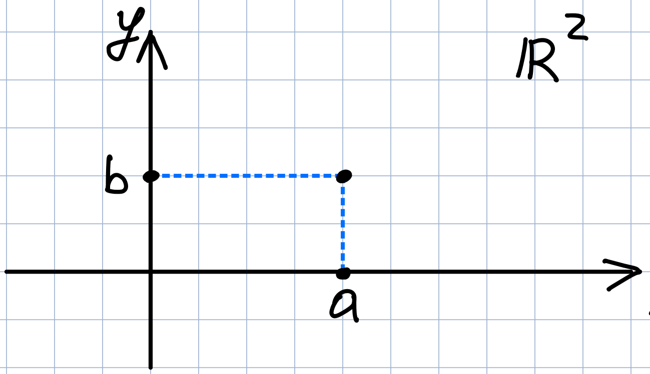
$$\oint_{\partial D}(L\,dx+M\,dy)=\iint_D\left(\frac{\partial M}{\partial x}-\frac{\partial L}{\partial y}\right)dA$$

# Three-dimensional coordinate system

**Reminder:** A point on a plane is given by a pair of numbers

$(a, b)$

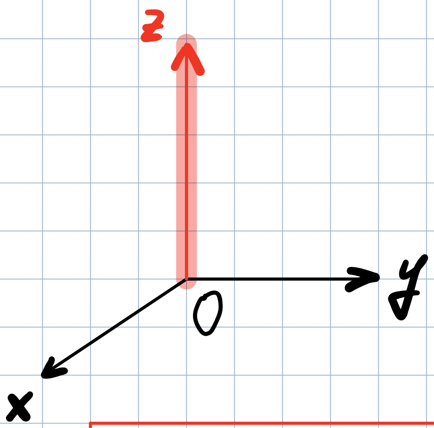
x-coordinate y-coordinate



- A point in space ($\mathbb{R}^3$) is given by triple of numbers

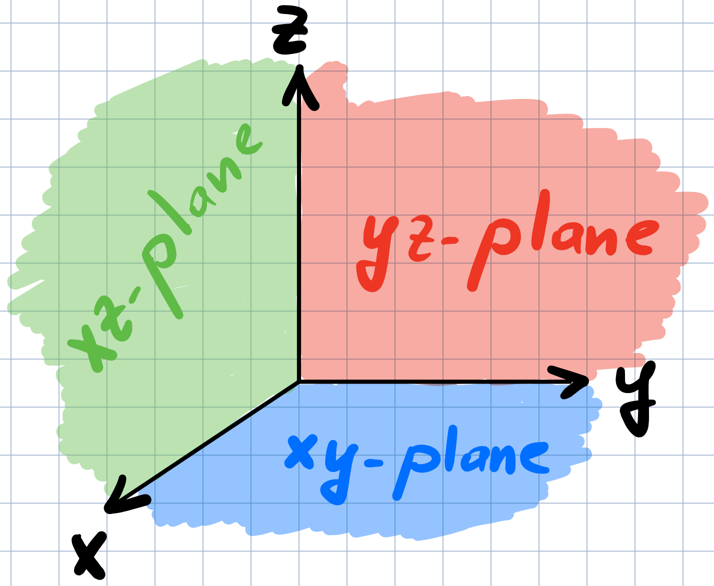
$(a, b, c)$

x-coord. y-coord. z-coordinate



the direction of the z-axis is determind by the **right-hand-rule**

- Choose a fixed point 0 (the origin) and 3 directed, mutually perpendic. lines through 0:
  x-axis
  y-axis
  z-axis

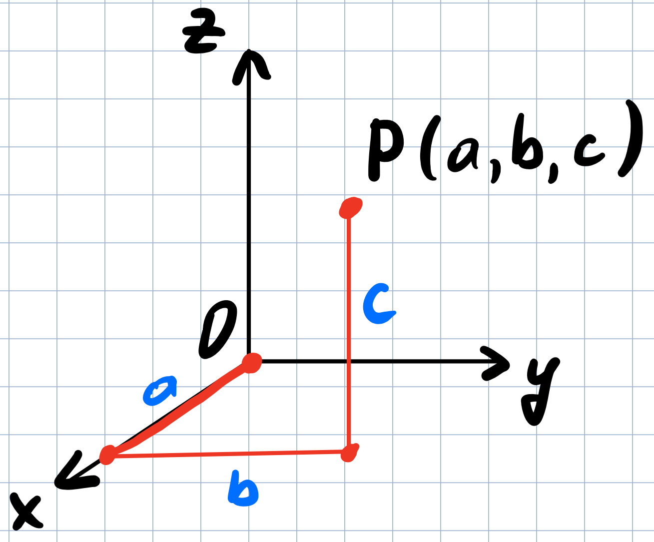


- Coordinate axes determine coordinate planes:

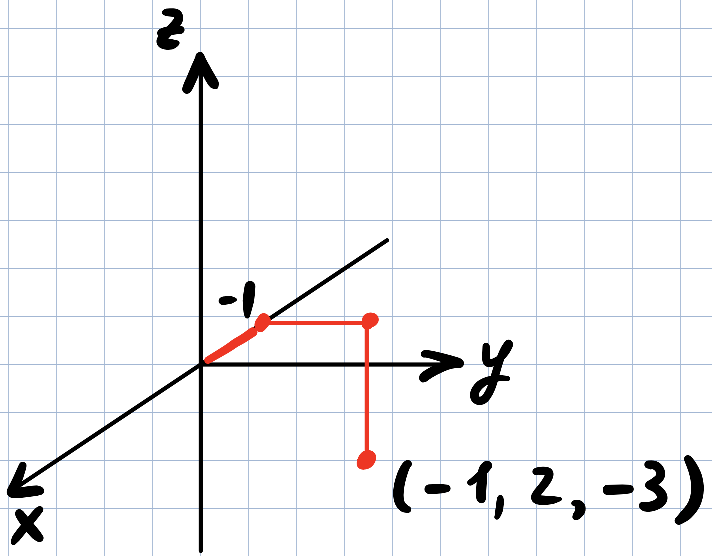
xy-plane
yz-plane
xz-plane

- Coordinate planes divide space into 8 octants

1st octant = all points (a, b, c) with $a, b, c > 0$

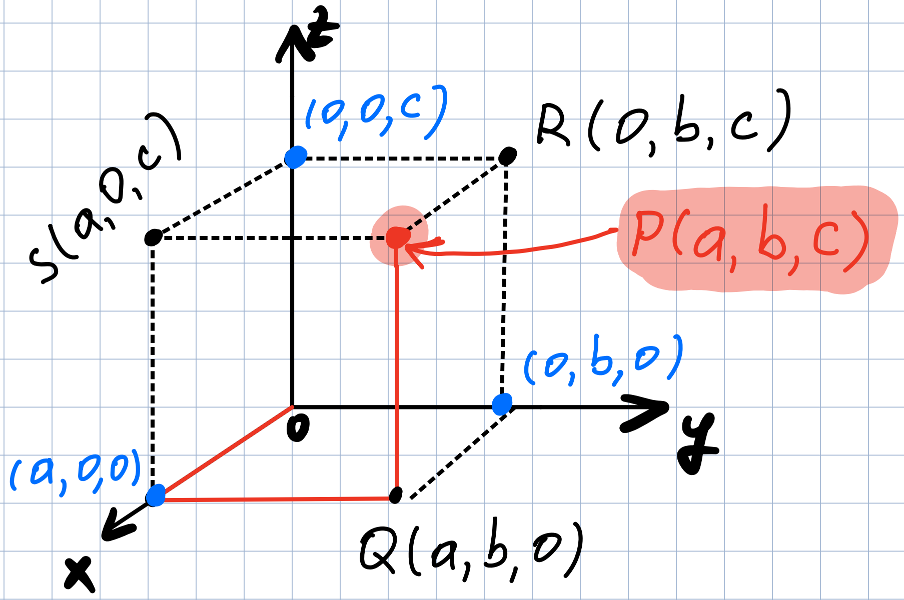


- $a$ is the (directed) distance from $P$ to the $yz$-plane
- $b$ is the (directed) distance from $P$ to the $xz$-plane
- $c$ is the (directed) distance from $P$ to the $xy$-plane



NB: coordinates can be negative!

The point $P(a, b, c)$ determines a rectangular box:



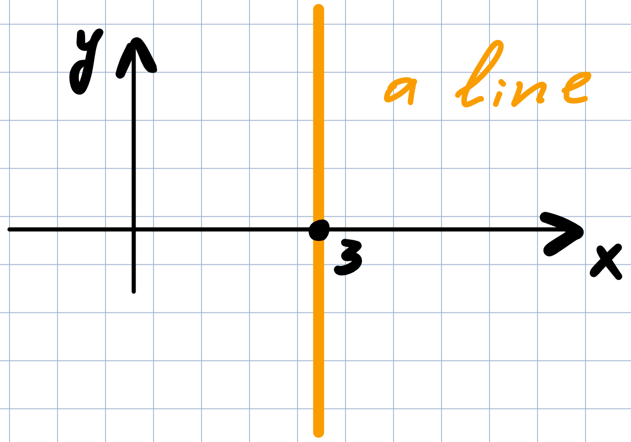
- $Q(a,b,0)$ is the projection of P onto $xy$-plane
- $R(0,b,c)$ is the projection of P onto $yz$-plane
- $S(a,0,c)$ is the projection of P onto $xz$-plane

# Surfaces

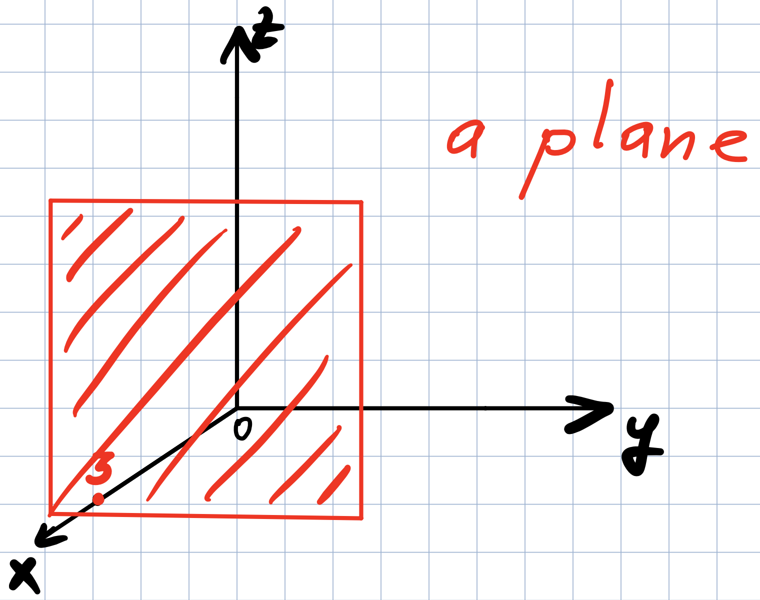
On $\mathbb{R}^2$, graph of an equation involving $x,y$ represents a curve in $\mathbb{R}^2$

In $\mathbb{R}^3$, an equation involving $x,y,z$ represents a surface in $\mathbb{R}^3$

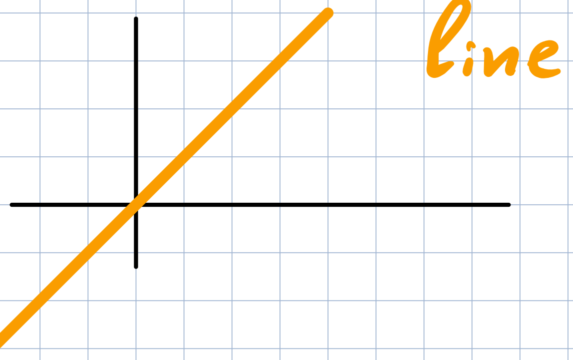
**Ex:** $\{(x,y) \mid x=3\}$



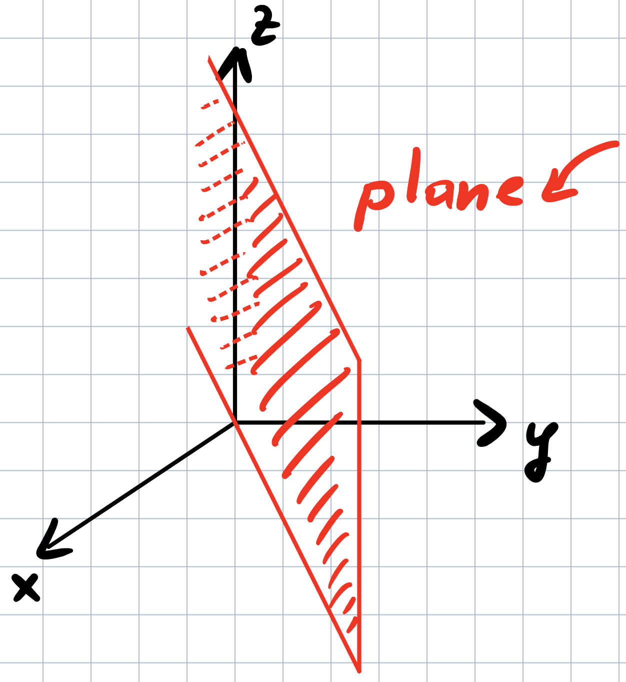
$\{(x,y,z) \mid x=3\}$



**Ex:** In $\mathbb{R}^2$: $x = y$

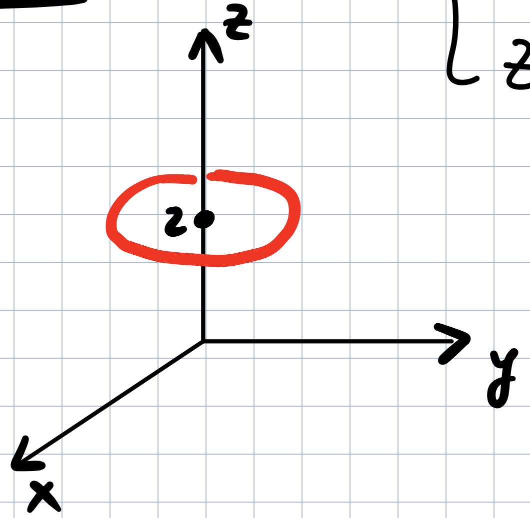


In $\mathbb{R}^3$: $x=y$

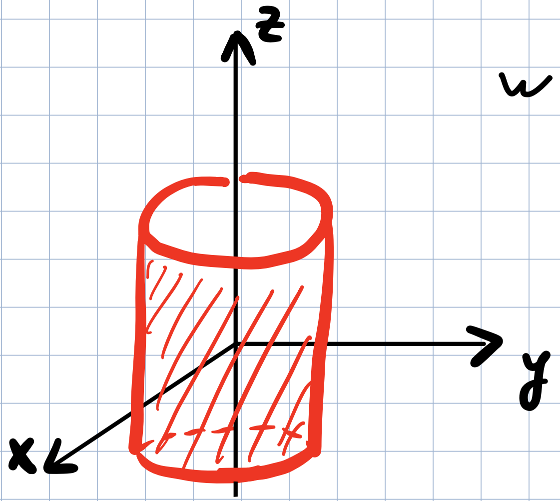


vertical (i.e. perpendicular to xy-plane) and intersecting it in the line $\begin{cases} z=0 \\ x=y \end{cases}$

**Ex:** (a) $\begin{cases} x^2+y^2=1 \\ z=2 \end{cases}$ circle of radius 1 centered on z-axis in the horizontal plane $z=2$



(b) $x^2+y^2=1$ - cylinder of rad. 1 whose axis is z-axis



# Distance:

On $\mathbb{R}^2$: distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$

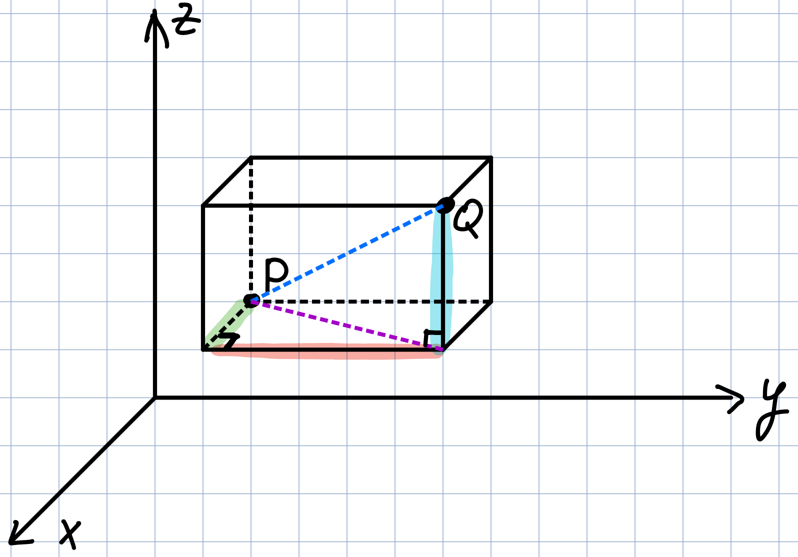
is

$$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

In $\mathbb{R}^3$:

$P(x_1, y_1, z_1)$ $\qquad$ $Q(x_2, y_2, z_2)$

$$\underline{|PQ|} = \sqrt{\underline{\underline{(x_2-x_1)^2} + \underline{(y_2-y_1)^2}} + \underline{(z_2-z_1)^2}}$$



**Ex:** $P(0, 1, -2)$ $\qquad$ $Q(1, 3, 0)$

$$|PQ| = \sqrt{(1-0)^2 + (3-1)^2 + (0-(-2))^2} = \sqrt{1+4+4} = 3$$

A sphere of radius $R$ with center $P(a, b, c)$ is given by equation

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2$$

**Rmk:** In case $P=0$: $x^2+y^2+z^2=R^2$

**Ex:** Show that the equation

$$x^2+y^2+z^2+4x-6y+2z+6=0$$

defines a sphere; find it's radius and center.

$$\underbrace{x^2+4x}_{x^2+2\cdot 2x+2^2}+\underbrace{y^2-6y}_{y^2-2\cdot 3y+3^2}+\underbrace{z^2+2z}_{z^2+2z+1^2}=-6$$

$$(x+2)^2+(y-3)^2+(z+1)^2=4+9+1-6$$

$$(x-(-2))^2+(y-3)^2+(z-(-1))^2=8$$

$$P(-2, 3, -1) \qquad R=\sqrt{8}=2\sqrt{2}$$

**Ex:** Which of the points $P(4,-2,6)$ and $Q(-6,-3,2)$ is closest to yz-plane?

**Sol.:**

1) The shortest distance between any point and any of the coordinate planes will be the distance between that point and its projection onto that plane.

2) Let's call the projections of P and Q onto the yz-plane $\bar{P}$ and $\bar{Q}$ respectively. Then

$$\bar{P}=(0,-2,6) \quad \text{and} \quad \bar{Q}=(0,-3,2)$$

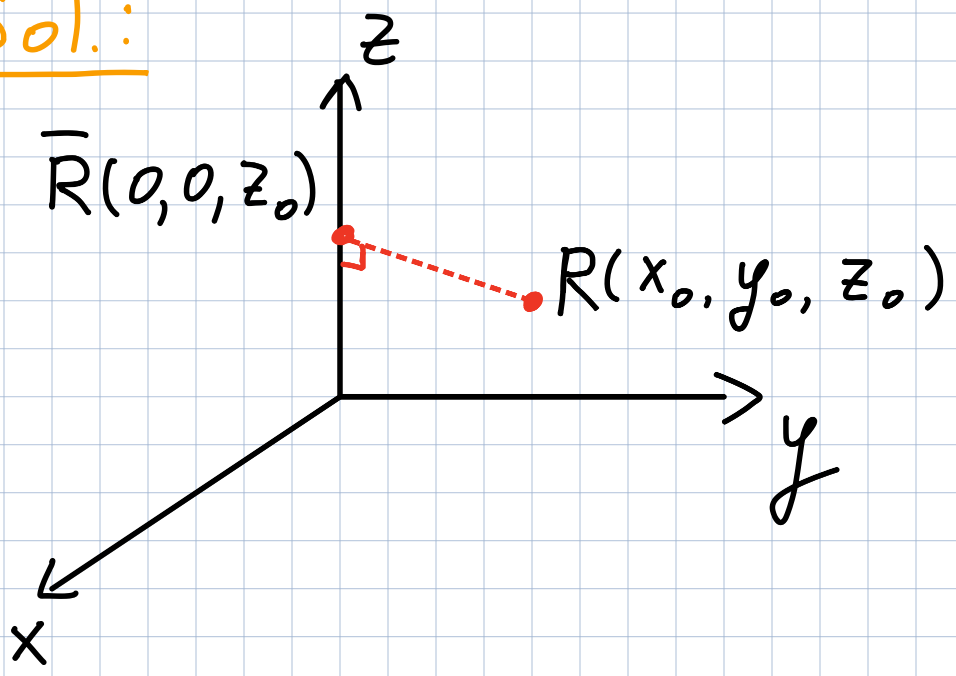
3) $$|P\bar{P}|=\sqrt{(4-0)^2+(-2-(-2))^2+(6-6)^2}=4$$

$$|Q\bar{Q}|=\sqrt{(-6-0)^2+(-3-(-3))^2+(2-2)^2}=6$$

$\Rightarrow$ P is closest to yz-plane.

**Ex:** Which of the points $P(-1,4,-7)$ and $Q(6,-1,5)$ is closest to the z-axis?

**Sol.:**



$$|R\bar{R}| = \text{dist}(R, z\text{-axis})$$

1) $\bar{P} = (0,0,-7)$

$\bar{Q} = (0,0,5)$

2) $|P\bar{P}| = \sqrt{(-1-0)^2 + (4-0)^2 + (-7-(-7))^2} = \sqrt{17}$

$|Q\bar{Q}| = \sqrt{(6-0)^2 + (-1-0)^2 + (5-5)^2} = \sqrt{37}$

3) $\Rightarrow$ P is closest to the z-axis.